STATEMENTS AND METHODS OF SOLVING INVERSE PROBLEMS OF RADIATIVE HEAT EXCHANGE

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Results are presented of analyzing statements and methods of solving inverse problems in the theory of radiative heat transfer.

A new scientific and technological direction, related to the study of inverse problems of heat transfer (IPHT), has been formulated and developed intensely in the last decade within the general theory of heat transfer. Mostly investigated was the class of inverse problems of thermal conductivity (IPTC), whose regions of practical application and methods of solution were considered in numerous studies, including the monographs [1-4]. The close attention to problems of precisely this class of IPHT is explained, in particular, by the wide use in experimental investigations of temperature measurements in solids by means of thermocouples. In projecting and optimizing parameters of technological systems with heat transfer dominated by radiation, in experimental determination of temperature fields and optical properties by information on radiation and in other practically important cases problems are generated, being a class of inverse problems of radiative heat transfer (IPRHT). The scientific and technological areas of practical application of IPRHT are extremely diverse and are determined by their primary value for applied solar energy [5, 6], astrophysics [7], IR heating and drying technology [8, 9], metallurgy and energetics [10-13], cosmonautics [14, 15], high-temperature physics [16] and technology [17], etc. Below we analyze the statements and methods of investigating this class of IPHT.

The earliest study devoted to the problem under consideration is, obviously, [18], where the term "inverse problem of radiation" was also introduced.

The basis of casting IPRHT self-consistently consists of the foundation of three autonomous sections of the general theory of heat transfer: thermal conductivity, convection, and radiation. The reasons are covered by the substantial difference in the heat transfer mechanism, which in turn leads to a different mathematical description of the corresponding problems. Thus, integral equations are most widely used to describe radiative heat transfer, while the description of thermal conductivity processes is based on differential equations. This implies substantial differences in the methods used to study radiation and thermal conduction processes [19].

The equations describing the heat transfer process establish a causal relation between its characteristics (parameters). The purpose of the direct problems of heat transfer is finding the characteristic actions from the causal characteristics. If by some information it is required to determine deficient characteristics, we handle some statement of the inverse problem [1, 2].

In an arbitrary case, in stating the inverse problem (IP) within some mathematical model the statement of the direct problem, with respect to which one can formulate some set of inverse problems, is assumed known. Consequently, generally speaking, the diversity of statements of inverse problems is wider than that of direct problems.

The importance of developing corresponding investigation methods in the region of inverse problems of heat exchange of all classes is primarily due to the necessity of solving many practical problems during the process of thermal projection and modeling operating regimes of complex energy saturation machines and aggregates, in creating which complex computational—experimental investigation methods play an important role. It is precisely this region of contemporary machine construction which stimulated the trend of developing methods introduced into practice and developing the methodology of IPHT as a whole [20-22]. The

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necessity of solving IPRHT is naturally generated when radiative heat transfer in the technological system investigated is predominant and is subject to the characteristics of the process (system), on which depend the resulting (output) parameters of radiation transport. In particular, a wide spectrum of IPRHT is generated during exhaustion of thermal construction regimes on stations of radiative heating [23]. This is primarily an identification problem of structures and parameters of mathematical models of real thermal exchange processes, synthesis and optimization of projective constructions and parameter regimes of radiative heating instruments, interpretation of experimental data on indirect measurements of the quantities characterizing heat transport by radiation. It is noted that in vacuum the heat transfer between bodies is realized excusively by radiation, due to which the solution of a whole range of problems, related to the calculation of external heat transfer of cosmic instruments and their thermo-vacuum experiments [21, 22], must necessarily be based on IPRHT methods. The experimental determination of parameters of radiative heat transfer from information concerning radiation is also often related to the necessity of solving IPRHT. In this case the same heat transfer parameters can be found both from IPTC and IPRHT (for example, extents of blackness or angular coefficients). The belonging of an inverse problem to some class is determined by the relevant information for searching unknown quantities: If the temperature is known, we deal with IPTC [24], while if the radiative flux is known — we deal with IPRHT [16, 25, 26].

Consider in more detail the features of statements and methods of solving IPRHT. Methodologically this is advisable to do within the simplest and well-studied diffusely gray model of heat transport by radiation, which has so far been widely used to solve practical engineering problems [27, 28].

For the given case the driving characteristics of heat transfer by radiation are temperature, optical properties and the surface geometry of the radiating system, and the characteristic consequence — the surface density of radiation flux (primarily the incident E_{inc}, the resulting E_{res}, and the effective E_{ef}). The intrinsic radiation flux, whose value is directly determined by the temperature and optical properties, can be assumed to be a complex driving characteristic. Depending on the driving charactersitic, we distinguish between temperature, optical, and geometrical IPRHT (naturally, combinations of these statements are also possible).

We note that the problem of searching parameters for a given radiating system on two parts of the total surface corresponding to the temperature and to E_{res} relates to the mixed statement of the problem of radiative heat transfer [29]. Below we assume that without exception all problems of finding several causal characteristics are independent of whether the characteristic consequences are known and given on part or the whole region of their definition.

As is well-known, mathematically many inverse problems, including heat exchange, are often incorrectly stated [1]. The nature of incorrectness of IPRHT depends on the specific physical content of its statement. Thus, the mixed statement of the problem of radiative heat transfer mentioned above for diffuse-gray surfaces is not always correctly stated: If E_{res} is given on the whole surface, this problem can have a manifold of solutions, while in the opposite case the existence condition corresponding to the physical meaning of the problem can be violated [16].

The incorrectness of IP is manifested primarily in the violation of continuous dependence of its solution on the input data, as occurs in the case of flux density of incident radiation given on part or the whole surface of the radiating system [18]. The mathematical incorrectness of the problem is due to the specific physical nature of the effect investigated, consisting of the fact that the individual features of the causal characteristics are consequently manifested in strongly smoothed form. For IPRHT the extent of similar smoothing can be traced on the example of the problem of radiative heat transfer in a spherical band. Therefore, for the given specific geometric configuration the angular coefficient between any two surface elements is a constant quantity (for a fixed radius of the sphere), and the general solution of the given problem for arbitrarily assigned temperature and optical property distributions is described in the form [27]

$$E_{ef}(M) = E_c(M) + \varepsilon^*(M)\overline{E}_c(M);$$
$$\overline{E}_c(M) = \frac{1}{F}\int_F E_c(M)\,dF_M; \quad \varepsilon^*(M) = \frac{1-\varepsilon(M)}{\varepsilon(M)}. \tag{1}$$

The flux densities of the incident and effective radiation are related by

$$E_{\mathrm{inc}}(M) = \int_F E_{\mathrm{ef}}(N)\, K(M, N)\, dF_N, \tag{2}$$

where K(M, N) is the kernel of the integral equation, completely determined by the geometric parameters of the radiating system (for a spherical band $K(M, N) = F^{-1}$ = const). Substituting (1) into (2), we obtain

$$E_{\mathrm{inc}}(M) = \overline{E}_e \left(1 + \frac{1}{F} \int_F \varepsilon^{*}(N)\, dF_N\right). \tag{3}$$

Thus, according to expression (3) $E_{\mathrm{inc}}(M)$ = const at each point of the internal surface of the sphere and is independent of the specific temperature distribution and the extent of blackness, including the case in which the extent of blackness and the temperature have nonvanishing values at one point. Similar smoothing (in the general case stronger than in thermal conductivity problems) is a consequence of the physical pattern of the flow process of heat transport by radiation, characterized by the multiple features of scattering (surface scattering in the given case) and energy reradiation.

To obtain steady (regular) solutions of many IP it is necessary to apply special methods, developed within the general theory of incorrect problems, acquiring recently intensive development mostly due to studies of Soviet mathematicians (see the references in [30]). A distinct feature of this theory is the reduction of the original incorrect statement of the problem to a conditionally correct statement by using some measure of a priori information (quantitative or qualitative) on the unknown solution and (or) the accuracy of the assigned original data. The possible statements and sequence of solutions of IPHT are considered in general form in [2].

Analysis shows that IPHT within the diffuse-gray model can be formulated mathematically in a double manner: in the form of a system of integral equations of kind I and II [31] (the integral statement) or as a problem of mathematical programming [32] (extremal statement).

Problems in many areas of mathematical physics reduce to integral equations of kind I, including astrophysics [76], gravimetry [33], plasma diagnostics [34], geophysics [35], heat transfer [1], thermal physics of semitransparent media [16], and others. For IPRHT within diffuse-gray models the solutions of integral equations of kind I are related to several problems of reconstructing the temperature fields on thermal benches with radiative heating [18] and the reproduction of temperature and (or) optical surface properties from measurement results by means of optical pyrometers in the presence of background radiation [16, 26, 31].

The use of the integral statement (the method of mathematical model inversion by the terminology of [2]) is restricted by the validity of mathematical models adopted for the analysis and their corresonding integral equations of kind I. For several types of these equations were developed steady (regular) methods and computer programs [7, 34, 36, 37], using which it is possible to solve a number of IPRHT, including the special case of the diffuse-gray approximation. However, even for this simplest physical model in the studies known by us problems of systematic investigation of practical application of methods of the theory of incorrect problems for IPRHT solutions of the given type are not considered. This position can be partially explained by the fact that a similar method (unlike many existing ones) of solving the integral equations considered must admit a representation of the kernel in tabular form (not analytically) with a nonuniform step (matrices of angular coefficients for zonal approximations of the problem), whose elements are determined numerically with some (usually unknown) error. The advantages of the integral statement of IPRHT (compared with the extremal) consist of the high economy in the method of solving partial statements of problems, such as temperature and optical ones. As mentioned in [28], to investigate the same geometric IPRHT the application of a variational approach is required.

The statement of IPRHT in extremal form is most universal, and is widely used in solving many inverse problems. The given approach to solving the inverse problem assumes that one knows: some functional, whose extremum corresponds to the unknown solution; an equation describing the heat transfer process by radiation in the system considered (usually appearing in implicit form in the purpose functional); and restrictions following from the physical essence of the various parameters, the requirement of technical assignment, usability conditions, etc. To the extremal statement belong naturally problems of designing many techno-logical instruments with predominant heat transfer by radiation, consisting in turn of the determination of such (optimal) projecto-constructive parameters, for which the system created will have best quality (cost, weight, economy, etc.).

It must be noted that in the absolute majority of studies devoted to optimization of some or other radiative heat transfer processes and instruments there exist no total algorithms of general search of the unknown solution (see, for example, [12, 13, 38]). A solution by the method of nonlinear programming (slipping admitted) is a quite general problem in optimal arrangement of cylindrical radiators over a planar heating plate with a uniform flux density distribution assigned on it of the incident radiation, as given in [32]. The given problem belongs to a class of problems of parametric optimization (in a finite-dimensional space), whose difficulty of solution increases sharply with increasing number of varying parameters. To the procedure of parametric optimization can also be reduced problems of functional optimization (in infinite-dimensional space) by representing the unknown function by a gridded analog or in the form of a linear combination of functions (primarily orthogonal polynomials, spline functions). The latter method ("parametrization of functions" [1]) makes it possible, in the presence of some a priori information, to select justifiably the most convenient system of approximation functions, substantially abbreviate the dimensionality of the optimization problem and, as a consequence, the computing time, and obtain a valid approximation to the solution of the original continuous problem. Incidentally, for the equations (not only linear) following from the integral IPRHT statement one can apply parametrization of the unknown function by a sequence determining the unknown coefficients from the solution of the problem of parametric optimization [16].

The statement of IPRHT in extremal form does not remove the problems of investigating its correctness. A characteristic manifestation of the incorrectness in the given case is the multiextremum of optimization problems of similar type. The most common approach to overcoming the difficulties generated in this case consists of using the corresponding regularizing procedures. In isolated cases the enhancement of probability of achieving a global minimum can be reached by selecting a rational method of optimization and its adjustment parameters (of the initial approximation, the step of varying unknown quantities, etc.), as well as by using further information by means of special restrictions of quantitative or qualitative nature, allowing to isolate a class of correctness for the problems investigated [30].

Exactly the same is required of a wide selection of rational minimization methods for solving a variety of extremal problems of radiative heat transfer. The problems noted have so far not been investigated sufficiently.

As shown by the analysis carried out, mostly investigated at the present time has been a class of inverse problems of thermal conductivity. It is precisely here that notable success has been achieved recently in the theory, and interesting practical results were obtained in solving many applied IP. Practical inquiries, primarily in regions related to the complex computational-experimental analysis of processes and systems in which heat transfer by radiation plays an essential role, stimulate the performance of systematic studies in methods of solving inverse problems of radiative (and combination) heat transfer. It can be assumed that at the present time IPRHT methods are already at the initial phase of formation. Their accelerated development and introduction to the practice of applied studies depends, in our opinion, on the solution of a number of initial problems, such as the development of economic and quite universal algorithms and application program packages for the solution of direct problems of radiative heat transfer, ranging over regions of rational use and accuracy of results obtained; carrying out special studies on the correctness of the various statements of IPRHT and methods of obtaining their stable solutions; parallel investigation and comparison of various IP statements and methods of their solution with the purpose of creating a problem-oriented mathematical structure for numerical computer solution of radiative heat transfer problems in the direct and inverse statements.

In conclusion we note that the development level of theory and methods of solving direct problems and the attempt of solving various classes of inverse problems of heat transfer made it possible to turn to the statement and practical solution of a number of IP of complex (combined) heat excahnge [39, 40], as well as associated problems [41]. This natural transition process to the study of more complex problems will require the integration of methodological principles and practical methods used in the solution of various special IPHT.

NOTATION

E_{inc}, E_{res}, E_{ef}, E_c, flux densities of incident, resulting, effective, and proper radiation; $\varepsilon(M)$, $T(M)$, extent of blackness and temperature in the vicinity of point M; F, surface (surface area); r, radius of the sphere; $\overline{E}_c$, mean integral value of E_c; $\varepsilon^*(M)$, extent of blackness (emissivity) function; K(M, N), kernel of the integral equation (2); M, N, points on the surface F; and dF_M, dF_N, elementary areas near the points M and N, respectively.

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STABILITY OF SOLVING OPTIMIZATION PROBLEMS OF PARAMETERS OF RADIATIVE HEATING DEVICES

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Results are given of a study of correctness of problems related to the determination of optimal parameters of radiative heating devices from a given flux field of incident radiation.

Computational-theoretical analysis of construction arrangement schemes of radiative heating devices (RHD) is one of the problems solved during the preparation of thermal tests of materials and structures on benches of a radiative heater [1]. Similar studies are required for the determination of such (optimal) RHD parameters (the spatial location of the emitter, the screen shape, etc.), for which the realized conditions of thermal loading on the surface of the tested product corresonds fully to the given one.

From the point of view of the theory of inverse problems (IP) of heat transfer the search for optimal RHD parameters belongs to the class of inverse problems of radiative heat transfer (IPRHT) of the projection type. Similar problems are naturally formulated as extremal, and to solve them one uses methods of the theory of mathematical programming [2]. A specific feature of inverse problems consists of the fact that in the general case they may be incorrect, as a consequence of which the condition of uniqueness and (or) stability of the solution with respect to small varying input data may be violated [3].

In [4] was realized a parametric optimization of RHD, consisting of three sources of radiation and a planar screen, by means of one of the direct methods of nonlinear programming, the method of sliding tolerance. In this case of special studies no verification was carried out of the correctness of the statement of the problem mentioned. The purpose of the present study is a more detailed analysis of the given problem.

For this we consider the geometric IPRHT, the unknowns in which being the vertical coordinates of radiative line sources, for which the distribution $E_{inc}(x)$ on a plate of infinite length and finite width is closest to a uniform distribution with a given density E_{inc}^0 = const. The distances between all radiators over horizontal directions were taken identical, while the peripheral radiators, independently of their total number, were located over the edges of the plate. The extent of nonuniformity of $E_{inc}(x)$ was characterized by the quantity

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